AFWAL-TR-85-4121

OVERLOAD EFFECTS IN SUSTAINED LOAD CRACK GROWTH IN INCONEL 718

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NOVEMBER 1985

Interim Report Covering Period January 1985 Through October 1985

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS AD-A162 759
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFWAL-TR-85-4121
6a. NAME OF PERFORMING ORGANIZATION University of Dayton Research Institute 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Materials Laboratory (AFWAL/MLLN) Air Force Wright Aeronautical Laboratories
6c. ADDRESS (City, State and ZIP Code) 300 College Park Drive Dayton OH 45469	7b. ADDRESS (City, State and ZIP Code) Wright-Patterson Air Force Base OH 45433
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AF Contract F33615-84-C-5051
8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 61102F; PROJECT NO.: 2302; TASK NO.: P1; WORK UNIT NO.: 01
11. TITLE (Include Security Classification) Overload Effects In Sustained Load Crack Growth In Inconel 718	
12. PERSONAL AUTHOR(S) T. Weerasooriya* and T. Nicholas**	
13a. TYPE OF REPORT Interim 13b. TIME COVERED FROM 1/85 TO 10/85	14. DATE OF REPORT (Yr., Mo., Day) November 1985 15. PAGE COUNT 25
16. SUPPLEMENTARY NOTATION *UDRI **AFWAL/MLLN (+ the Square root of π	
17. COSATI CODES FIELD / GROUP / SUB. GR.	18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) crack growth, fatigue, overloads, crack growth retardation, nickel base superalloy, elevated temperature

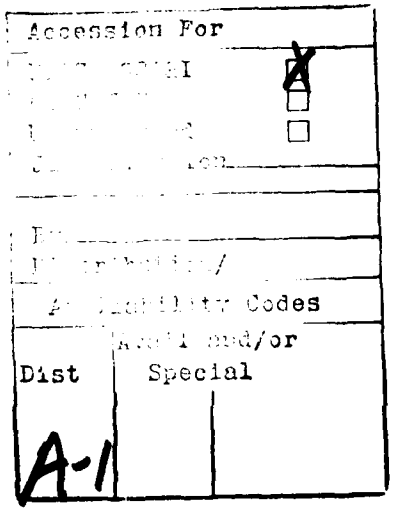
19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Crack-growth-rate experiments were conducted on CT specimens of Inconel 718 at 649°C. The loading spectrum consisted of a single 1 Hz cycle at R = 0.1 and a hold time. The ratio of the amplitude of the hold time to the maximum amplitude of the fatigue cycle, R_{in}, was 1.0, 0.9, 0.8 and 0.5 with hold times from 0 to 200 s. Tests were performed under computer controlled constant K conditions using values of the maximum of the fatigue cycle of 40 and 50 MPa·m$^{1/2}$. Data show that for R_{in} = 1.0, a linear summation model works well, while at R_{in} = 0.9 there is a measurable retardation effect on the crack growth during the hold time. For values of R_{in} less than 0.8, the sustained load crack growth is almost completely retarded. A simple retardation model is proposed which can fit the experimental data and is based on the concept of an overload plastic zone being produced by the fatigue cycles. It is concluded that hold times do not contribute to crack growth in this material unless their amplitude is at or near the maximum amplitude of the adjacent fatigue cycles.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☒ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Theodore Nicholas	22b. TELEPHONE NUMBER (Include Area Code) (513) 255-2689 22c. OFFICE SYMBOL AFWAL/MLLN

FOREWORD

Portions of work were performed under a contract with the Metals Behavior Branch (MLLN) of the Materials Laboratory, USAF Contract Number F33615-84-C-5051, under the administration of Dr Theodore Nicholas. This work was performed at the Materials Laboratory and supported, in part, by in-house Project 2302P102.



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SECTION 1

INTRODUCTION

The recent introduction of a damage tolerant design requirement for critical structural components in U.S. Air Force gas turbine engines has created a need for accurate crack growth predictions [1]. The Engine Structural Integrity Program (ENSIP) requires that a pre-existing flaw be assumed present in a component and that fracture mechanics analysis and testing demonstrate that such flaw will not grow to a catastrophic size during the lifetime of the engine. To meet this requirement, crack growth rates must be predicted under typical mission spectra. For components such as turbine disks and spacers in advanced fighter or attack aircraft, the loading spectrum will generally consist of large amplitude, low frequency fatigue cycles with interspersed hold times. The hold times have durations of tens of seconds with amplitudes anywhere from the maximum of the fatigue cycles to less than half that amount, depending on the type of aircraft and the mission.

Concurrent with a design philosophy change from low cycle fatigue initiation to a fracture mechanics damage tolerant approach, engine operating temperatures have increased steadily. This has led to the development of higher temperature nickel-base superalloys and a tendency to use these alloys under conditions where time-dependent material behavior becomes important. Thus, the effect of hold times in a design spectrum which results in crack growth during the hold times, has to be considered as part of the life prediction methodology. Although there has been substantial effort devoted to the study of elevated temperature sustained load crack growth in numerous materials [2-6], there has been little

research directed at crack growth under mission spectra, particularly when such spectra involve both sustained loads and fatigue cycles with sustained loads at different levels than the maximum load amplitudes of the fatigue cycles.

In a previous investigation, a linear cumulative damage model was applied for the prediction of crack growth rates under simple spectrum loading consisting of fatigue cycles and hold times [7]. For Inconel 718 at 649°C, a material which exhibits a significant amount of time-dependent behavior, it was found that the linear cumulative damage model worked well when hold times were at the same level as the maximum load of the fatigue cycle. Here, the algebraic summation of the fatigue crack growth rate as determined from constant load amplitude fatigue tests and the sustained load crack growth as determined from constant load crack growth tests provided excellent correlation with experiments where the two types of loading were combined. However, when the sustained load level was dropped to 75 percent of the fatigue maximum, the linear summation model broke down completely. The net growth observed experimentally appeared to be due only to the fatigue cycles, and the linear summation model grossly overpredicted the growth rate under the simple spectrum.

This investigation was undertaken to evaluate the range of validity of a linear summation model and to determine the contribution of hold-time loading in a simple spectrum consisting of combinations of fatigue cycles with hold times of various duration and amplitude. A simple analytical model is developed based on the concept that the sustained load crack growth is retarded by the application of the overload fatigue cycles.

SECTION 2

EXPERIMENTS

A series of crack-growth-rate experiments was conducted using the simple load spectrum depicted in Figure 1. The spectrum consists of a single fatigue cycle of frequency 1Hz, load ratio R=0.1 and maximum stress intensity factor K_m of either 40 or 50 MPa$\cdot m^{\frac{1}{2}}$. Hold times, t_H, of durations varying from a few seconds to several hundred seconds were applied at amplitudes given by $K = R_{in} \cdot K_m$ where R_{in} is the ratio of the amplitude of the hold-time to the maximum amplitude of the fatigue cycle. Values of R_{in} of 1.0, 0.9, 0.8 and 0.5 were used which correspond to overload ratios of 1.0, 1.11, 1.25 and 2.0, respectively, where the overload ratio is the ratio of K_m in the "overload" fatigue cycle to the baseline K of the hold-time load. All tests were conducted on CT specimens of width W = 40 mm and thickness b = 10 mm under computer controlled constant K_m conditions. The test procedure is described in detail in Reference 8. Crack lengths were determined from compliance readings taken with the aid of an extensometer during the unloading portion of the fatigue cycle.

The material used in this investigation was Inconel 718 at 649°C having a standard heat treatment given in Table 1. The material is identical to that used in several previous investigations where it has been shown that it exhibits significant amounts of time dependent behavior [7,8]. Several test conditions were evaluated on a single test specimen under constant K control. For each condition, the crack was grown until it could be unequivocally determined that the crack was growing at a constant rate. This was accomplished by continuously monitoring average growth rate

using a least squares linear regression fit to the crack length versus number of cycles data. When no statistically significant deviation from an average growth rate was obtained from additional applied cycles, the growth rate was defined as reaching a steady state value. This value was used in the plots of cycle block growth rate against total cycle time $T = t_H+1$ (see Figure 1). In most cases, the total crack extension used to establish a steady state growth rate under a given test condition was less than 1 mm and was typically of the order of 0.5 mm. Thus, over 30 test conditions could be evaluated on a single specimen, making it possible to duplicate and verify the growth rate for any given condition. Growth rates in a single specimen were found to be reproducible to an accuracy of 10 percent under a given load spectrum using K as the crack growth rate correlating parameter. Both the reproducibility of the data and the achievement of constant growth rates under constant K conditions demonstrate that K is an adequate correlating parameter for this material.

SECTION 3

EXPERIMENTAL RESULTS

Crack growth rate per cycle block, da/dN, was plotted as a function of total cycle time, T, for the cases of K_m = 40 and K_m = 50 MPa·$m^{\frac{1}{2}}$ in Figures 2 and 3, respectively. Note that a cycle time of 1 s. corresponds to pure fatigue cycling at 1 Hz with no hold time. The symbols in each figure represent the experimentally determined growth rates under constant K conditions for various values of R_{in}. The three solid lines in each figure represent, from top to bottom, the analytical predictions from a linear cumulative damage model for values of R_{in} of 1.0, 0.9 and 0.8, respectively. The linear model numerically adds the growth rate for a single cycle of 1.0 Hz and the contribution from sustained load crack growth, $\dot{a} \cdot t_H$, where $\dot{a}$ is the growth rate per unit time for the sustained load crack growth at a stress intensity factor value K_s. Values of a as a function of K were determined from a parallel investigation using the same batch of material [9]. Several constant load tests were conducted to obtain crack length versus time data which were reduced to a versus K and fit with a sigmoidal equation to provide numerical values of a for any K value.

The data in Figures 2 and 3 reconfirm the previous observation that a linear cumulative damage model works well for R_{in} = 1.0, ie when the hold-time load is at the maximum value of the fatigue cycle. For values of R_{in} = 0.9, the linear model overpredicts by more than a factor of 2, thus there appears to be some retardation of the sustained load crack growth. For values of R_{in} = 0.8, there does not appear to be any contribution of the hold-time loading for values of hold times up to 200 s, the maximum

used in this investigation. The linear model grossly overpredicts the growth rate for this case. The same observation can be made for $R_{in} = 0.5$. For this latter case, results were obtained only for the first set of experiments where $K_m = 40$ MPa$\cdot$m$^{\frac{1}{2}}$. There was no reason to repeat these very time consuming experiments for $K_m = 50$ MPa$\cdot$m$^{\frac{1}{2}}$.

SECTION 4

ANALYTICAL MODEL

A simple model is developed to predict the crack growth rates under the simple spectrum of Figure 1. The model uses a linear cumulative damage concept by summing the contributions of the single fatigue cycle and the hold-time load. The fatigue cycle is assumed to retard the sustained load crack growth because the latter occurs within an enlarged or overload plastic zone due to the fatigue cycle. This retardation concept, similar to that used by Wheeler [10], is illustrated schematically in Figure 4. The plastic zones due to the sustained load of stress intensity factor K_s and the overload fatigue cycle of maximum stress intensity K_m have radii denoted by r_p and $\bar{r}_p$ with $\bar{r}_p \geq r_p$ since $K_s/K_m \leq 1$. If the overload is applied when the crack length is a_o, the tips of the plastic zones due to the baseline (hold-time) load and fatigue overload are at a_1 and a_2, respectively. For a crack advance of Δa from a_o, the crack will be growing in an overload plastic zone until $\Delta a = a_2 - a_1$. During this time, the growth rate will be retarded. The amount of retardation can be expressed by an effective value of stress intensity factor, K_{eff}, which is smaller than the applied K. Using a single-valued function of $\dot{a}$ versus K to represent the sustained load growth rate, a value of K_{eff} less than the applied K will result in a lower growth rate. For modeling purpose, K_{eff} is taken in the form

$$K_{eff} = K_s[1 - \alpha \exp(-\beta \Delta a)] \qquad (1)$$

where α and β are parameters to be determined and Δa is the amount of crack extension after application of the overload . The parameter β is chosen such that $K_{eff} \rightarrow K_s$ when $\Delta a = \bar{r}_p - r_p$, ie steady state crack growth resumes when the crack has traversed the overload plastic zone. This is accomplished by setting

$$\beta \Delta a = \pi\sqrt{2} \tag{2}$$

when $\Delta a = \bar{r}_p - r_p$, resulting in a value of K_{eff}/K which is within one percent of unity as shown in Figure 5. The plastic zone sizes are taken to be the plane stress values given by

$$\bar{r}_p = \frac{1}{\pi}\left(\frac{K_m}{\sigma_y}\right)^2 \quad , \quad r_p = \frac{1}{\pi}\left(\frac{K_s}{\sigma_y}\right)^2 \tag{3}$$

where σ_y is the uniaxial tensile yield stress of the material. Combining (2) and (3) provides an expression for β:

$$\beta = \frac{\sqrt{2}\pi^2\sigma_y^2}{K_s^2(\gamma^2-1)} \tag{4}$$

where the overload ratio γ is defined as

$$\gamma = \frac{K_m}{K_s} = \frac{1}{R_{in}} \tag{5}$$

It is important to note that β is fully defined from the test conditions and material properties and cannot be used as an adjustable parameter to fit experimental data. Thus, eq. (1) has only one parameter, α, which can

be chosen to fit data. Referring to Figure 5, it can be seen that the value of α determines K_{eff} immediately after the overload application and, therefore, can be used to model the value of sustained load crack growth rate.

SECTION 5

DETERMINATION OF CONSTANTS

Analytical predictions were made using a linear cumulative damage model which incorporates the retarded sustained load crack growth behavior given by eq. (1). Computations of crack extension, Δa, were made by numerical integration of da/dt using 1 s time increments for Δt. The crack growth rate, da/dt, is expressed in the form of a modified sigmoidal equation fit to experimental sustained load crack growth data [9]. The stress intensity K_s in the sigmoidal equation is replaced by K_{eff} from eq. (1). The only parameter which had to be determined was α and this was accomplished by trial and error by choosing a range of values for α. The results of these trials are shown in Figures 6 and 7 for K_m = 40 and K_m = 50 MPa·m$^{\frac{1}{2}}$, respectively, and for the case of R_{in} = 0.9. The plots for α = 0 represent the linear summation model for the case of no retardation. It can be seen that values of α = 0.15 and α = 0.27 fit the experimental data for the two cases very well, respectively. It is noted in both cases that the model overpredicts the growth rates for very short hold times below approximately 10 s.

For the case of R_{in} = 0.8, there appeared to be almost complete retardation for hold times up to the maximum of 200 s used in the experiments. In this situation, α could not be determined accurately as can be seen in Figure 8. Complete retardation is equivalent to a zero growth rate which corresponds to a value of $K_{eff} = K^*$ which is the threshold value of K in the å versus K representation of the hold-time data. In our modeling of the data using a modified sigmoidal equation, K^* = 24 MPa·m$^{\frac{1}{2}}$. Values of α = 0.25 and α = 0.4 represent complete retardation for the cases

of K_m = 40 and K_m = 50 MPa·$m^{\frac{1}{2}}$, respectively. To represent the data in Figure 8, values of 0.246 and 0.392, respectively, were chosen.

Calculations were made of the overload plastic zone size, defined as $\bar{r}_p - r_p$, through which distance a crack must grow under sustained load before it returns to its steady state growth rate. For the four cases corresponding to the two values of K_m and for R_{in} = 0.8 and 0.9 for each case, values of this overload zone size ranged from approximately 0.1 to 0.5 mm. Note in Figures 2 and 3 that all of the experimentally measured growth rates per cycle block were well below these values. Thus, for the simple spectrum used, the sustained load crack growth was always within the overload plastic zone and occurred at a reduced rate due to retardation. In fact most of the sustained load crack growth occurs for small values of $\Delta a/(\bar{r}_p - r_p)$, or near the origin of the x-axis shown in Figure 5. For these experimental conditions, the parameter α is the primary factor governing the growth rate in the analytical model. The parameter β, and the shape of the function K_{eff}/K_s shown in Figure 5, are of secondary importance for short hold times under approximately 100 s.

SECTION 6

DELAY TIMES

In studies of fatigue crack growth retardation due to overloads, the concept of a number of delay cycles is commonly used. Here, we introduce the equivalent concept of a delay time for sustained load crack growth. Figure 9 presents plots of the calculated value of Δa_s, the growth during the sustained portion of the load, as a function of the hold times, t_H, for the cases where $R_{in} = 0.9$. Also shown in dashed lines is the crack extension that would occur if there were no retardation. The delay times, or the net gain in time because of the retarded growth rates, are approximately 200 s. as shown in Figure 9. Delay times of approximately 4000 and 8000 s were calculated for the cases where $R_{in} = 0.8$ and $K_m = 40$ and 50 MPa$\cdot m^{\frac{1}{2}}$, respectively. In a parallel investigation on the same material, delay times of approximately 4000 and 20000 s were experimentally measured for $R_{in} = 0.83$ and $R_{in} = 0.67$, respectively, for similar values of K_m [9].

SECTION 7

DISCUSSION AND CONCLUSIONS

For the simple spectrum consisting of a fatigue cycle and a hold time, for the material used in this investigation, the hold time does not contribute to crack growth except when its amplitude is close to that of the maximum of the fatigue cycle. If the sustained load crack growth is considered to be retarded by the single cycle overload, it appears that overload ratios greater than approximately 1.2 severely retard growth for hold times as long as 10^3s.

A single parameter retardation model for sustained load crack growth combined with a linear summation model which adds cyclic and (retarded) sustained load contributions appears to be adequate for predicting crack growth rates under the simple load spectrum used in this investigation. The model correlated data covering a range in hold times up to 200 s, although the model was slightly conservative for short hold times up to approximately 10 s. It appears that in addition to the retardation of the sustained load crack growth due to a fatigue overload, there is also a slight retardation of the fatigue crack growth due to the sustained load. This latter phenomenon was not modeled in this investigation.

Most of the retardation modeled in these experiments covered short hold times corresponding to small crack extensions compared to the size of the overload plastic zone. Thus, the initial growth rates directly after an overload appear to be modeled well. In spite of the fact that overloads are repetitively applied before the crack has grown even a small distance through the overload plastic zone, a steady state growth rate was achieved in all experiments under constant K conditions. This provides further

evidence for the use of K as a correlating parameter for sustained load crack growth in this material.

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TABLE 1

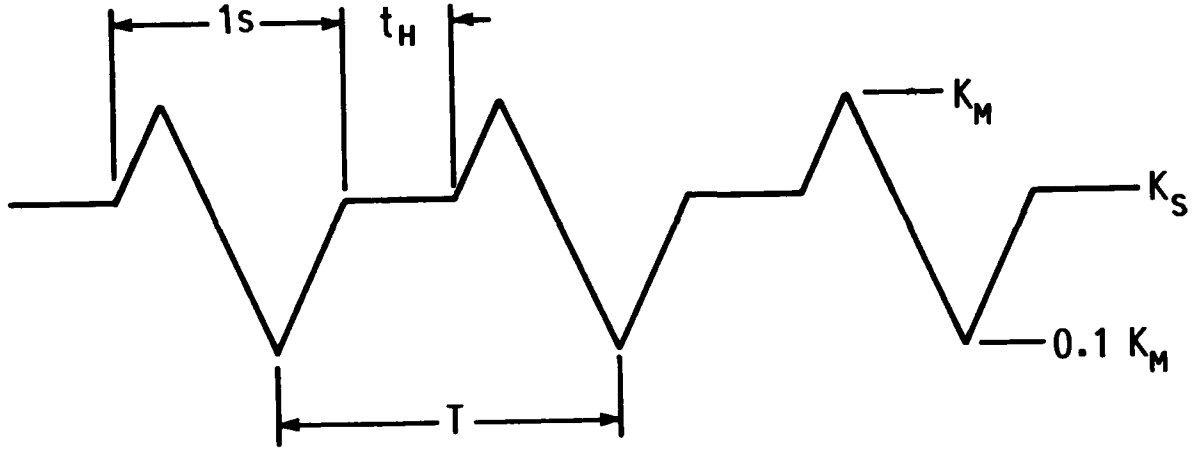
HEAT TREATMENT FOR ALLOY 718

STEP 1: Anneal at 968°C (1775°F) for 1 Hour, Then Air Cool to Temperature

STEP 2: Age Harden at 718°C (1325°F) for 8 Hours, Then Furnace Cool at 56°C/Hr (100°F/Hr) to 621°C (1150°F)

STEP 3: Age Harden at 621°C (1150°C) for a Total Aging Time in Step 2 and Step 3 of 18 Hours

STEP 4: Air Cool to Room Temperature



$$R_{IN} = \frac{K_S}{K_M}$$

Figure 1. Loading spectrum consisting of 1 Hz fatigue cycle and sustained load.

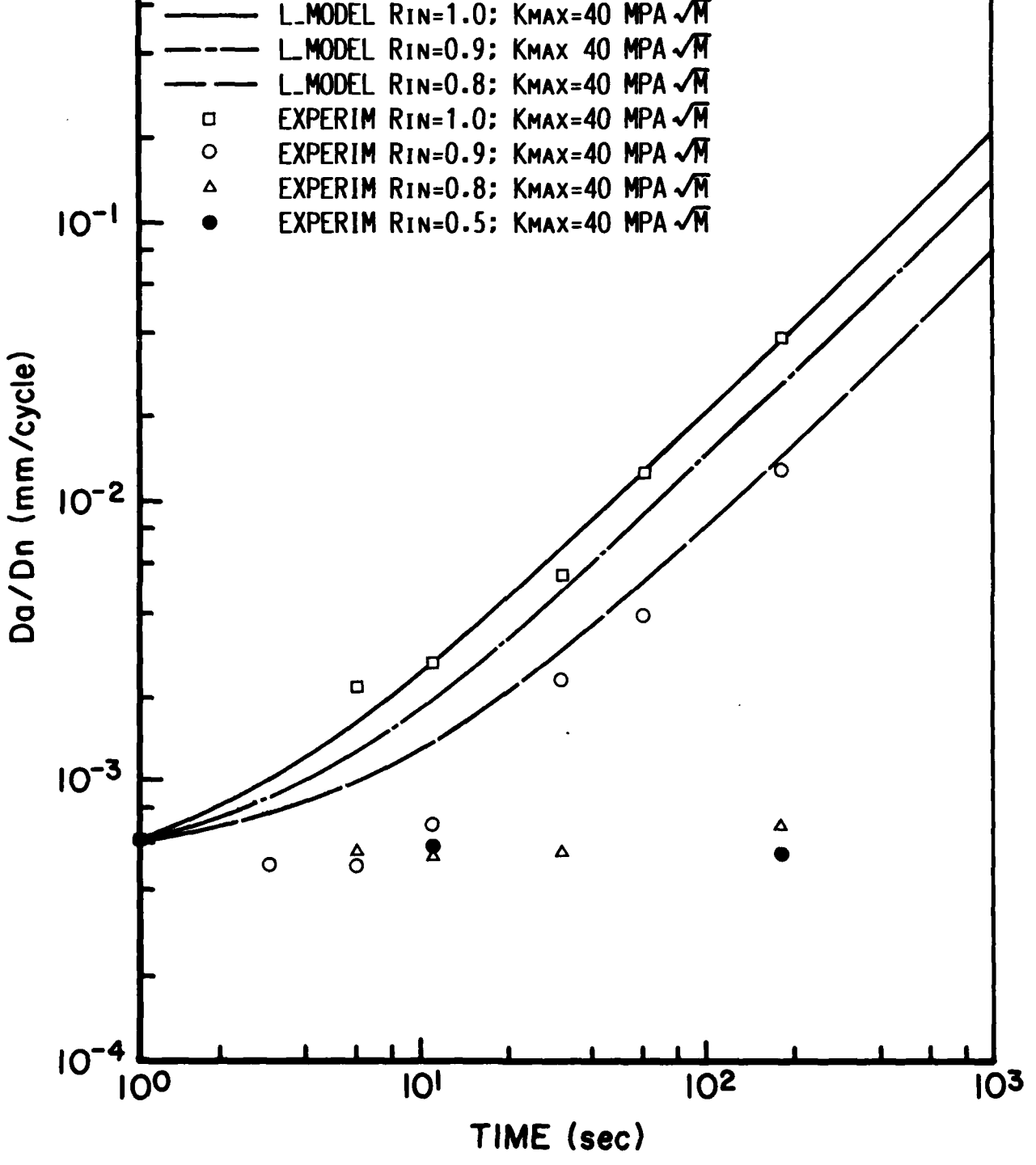


Figure 2. Crack growth rate data plotted against total cycle time for K_{max} = 40 MPa·m$^{\frac{1}{2}}$ as a function of different R_{in} values. Linear model data is also given in the figure.

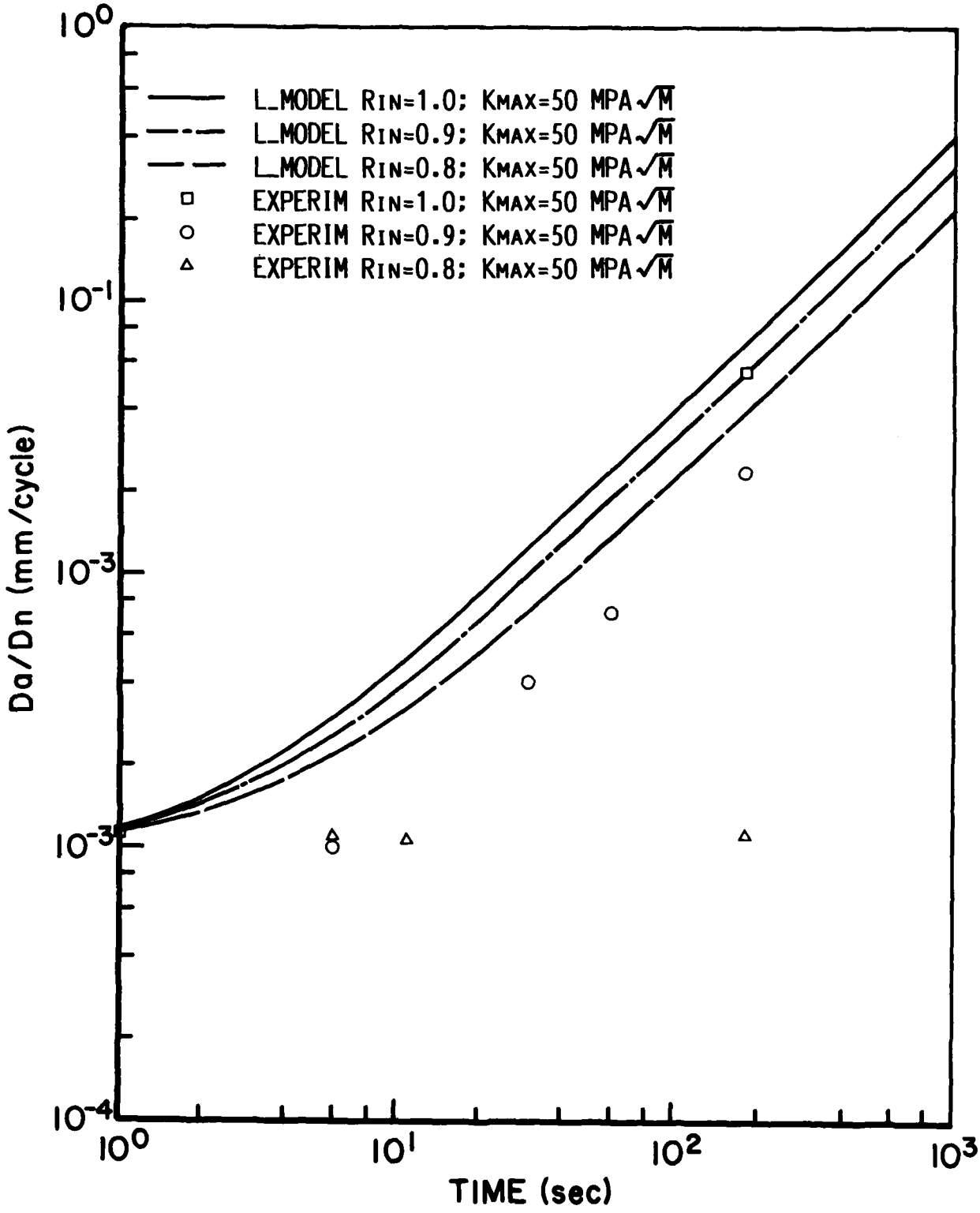


Figure 3. Crack growth rate data plotted against total cycle time for K_{max} = 50 MPa·m$^{\frac{1}{2}}$ as a function of different R_{in} values. Linear model data is also given in the figure.

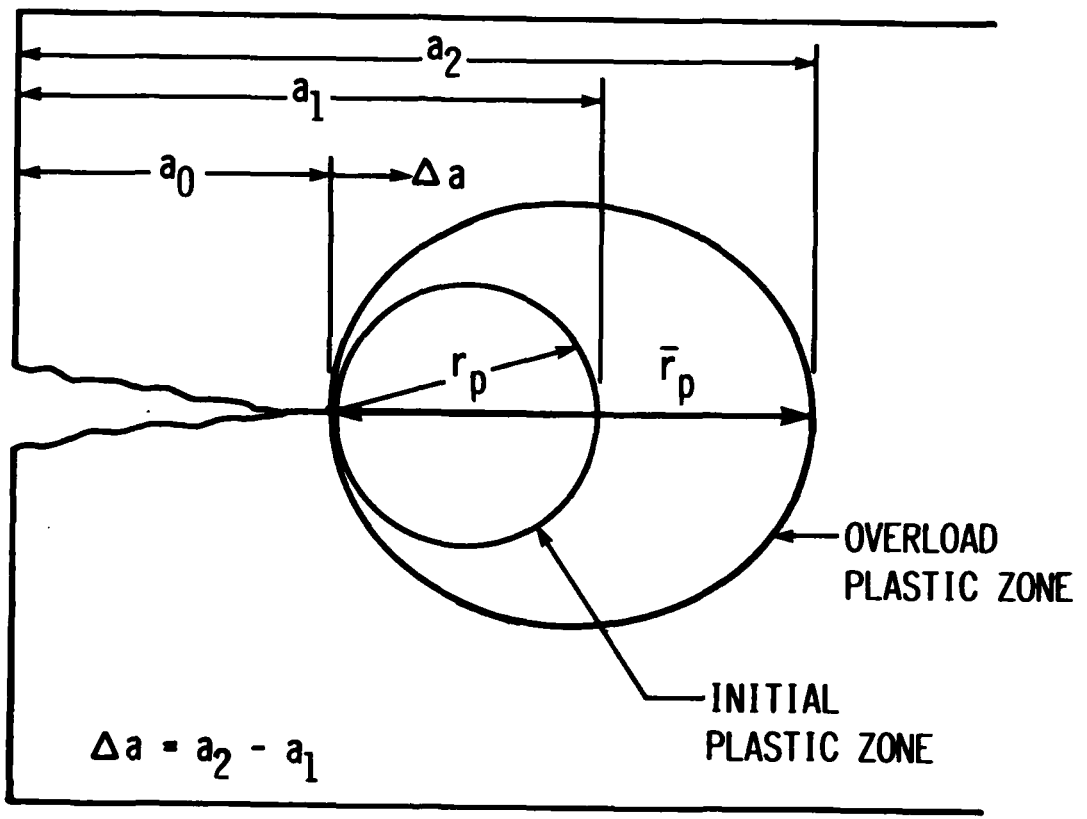


Figure 4. Schematic diagram of the plastic zones at the crack tip.

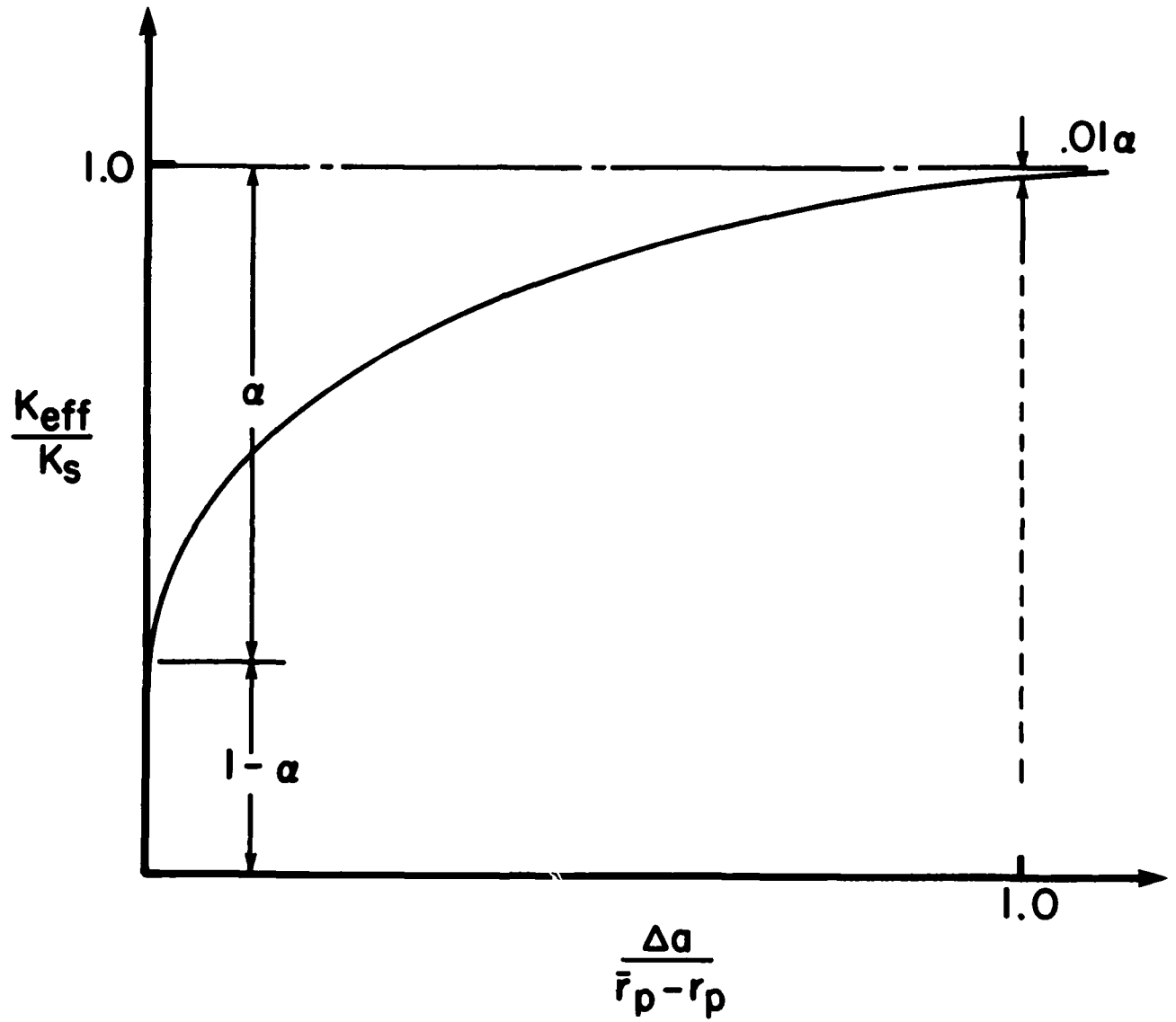


Figure 5. Variation of K_{eff}/K_s as function of normalized crack advance from the crack tip in the overload plastic zone.

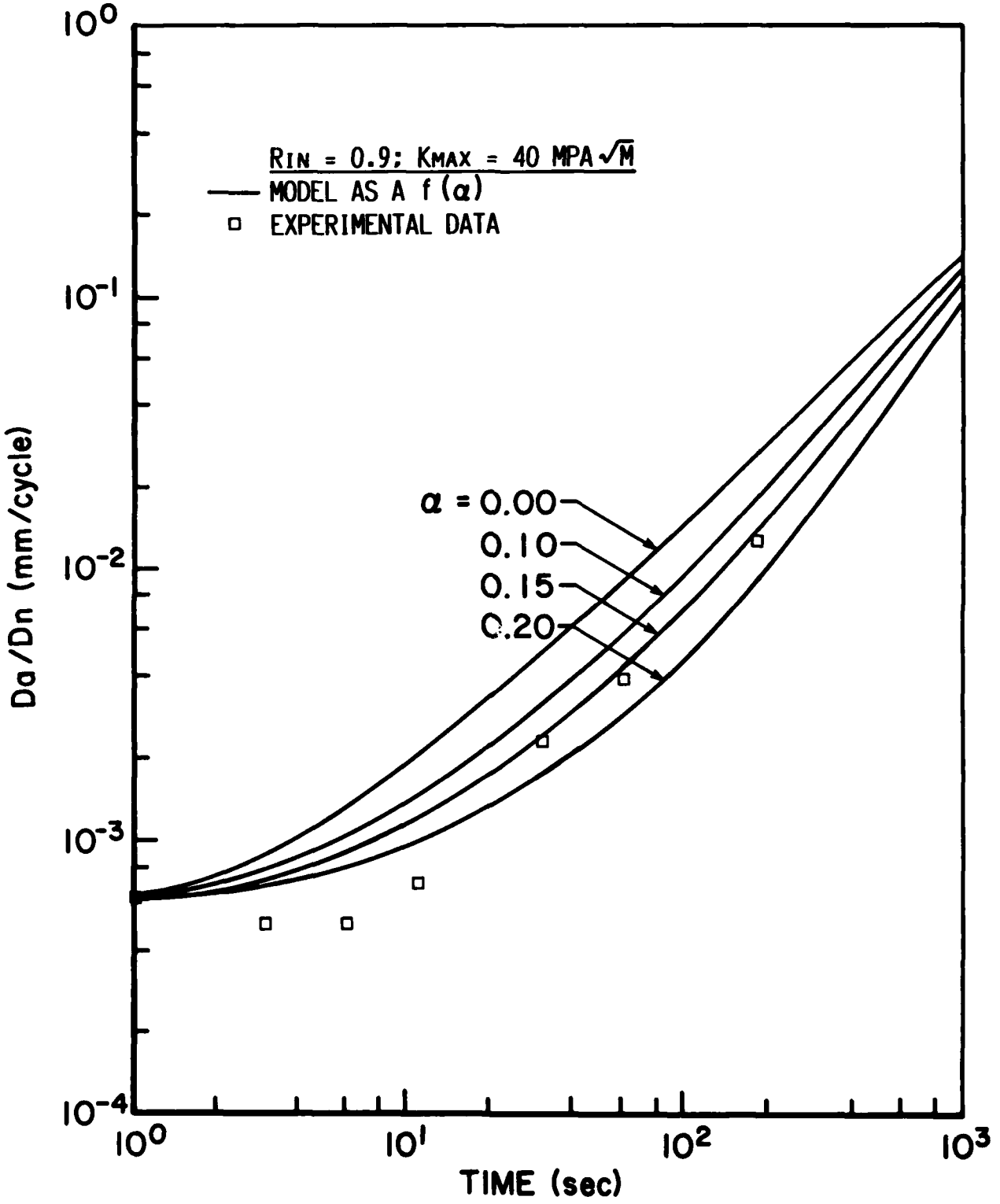


Figure 6. Calculated crack growth for different values of α for K_{max} = 40 MPa·m$^{\frac{1}{2}}$ and R_{in} = 0.9. Data are also shown in the figure.

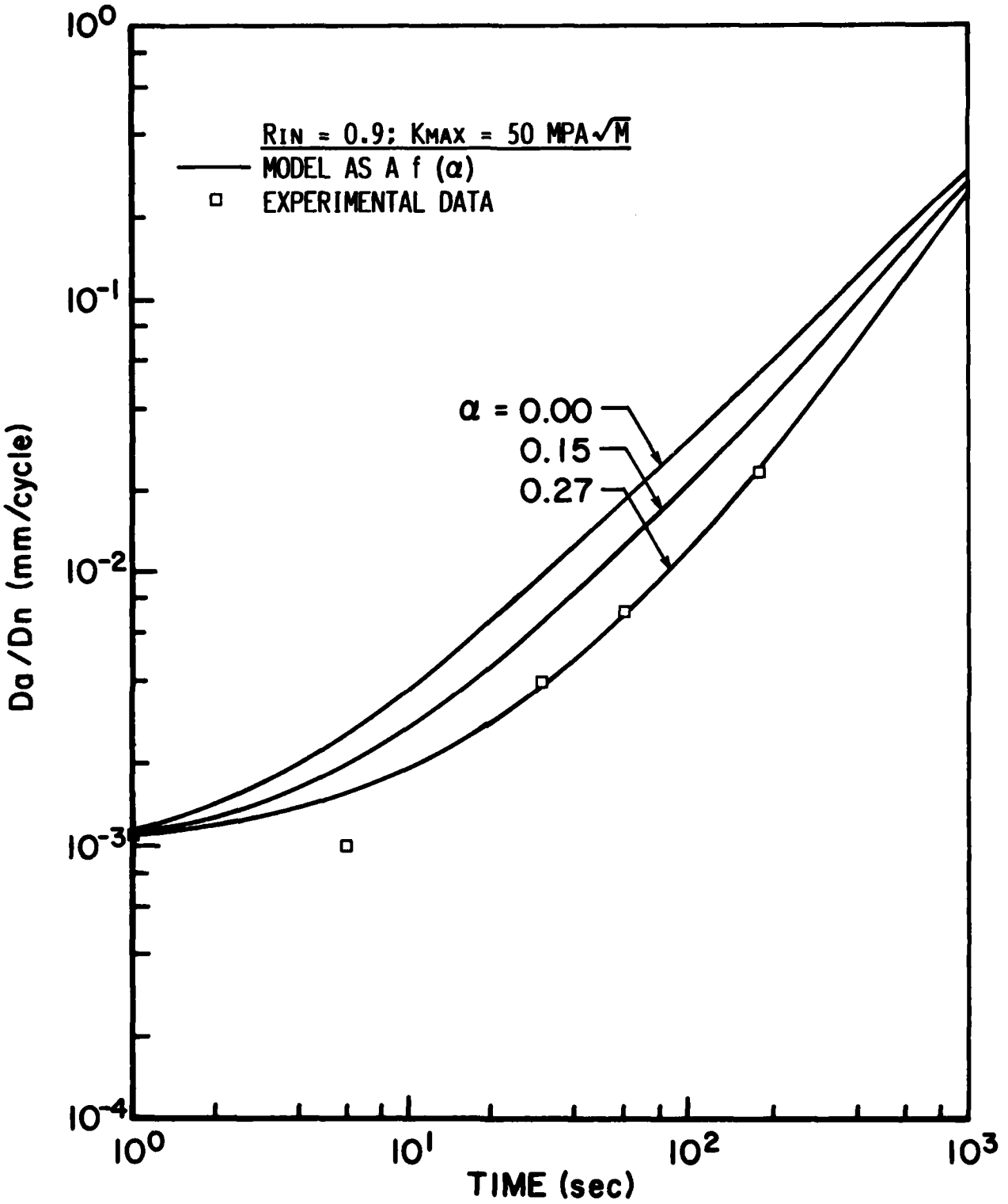


Figure 7. Calculated crack growth rate for different values of α for K_{max} = 50 MPa·m$^{\frac{1}{2}}$ and R_{in} = 0.9. Data are also shown in the figure.

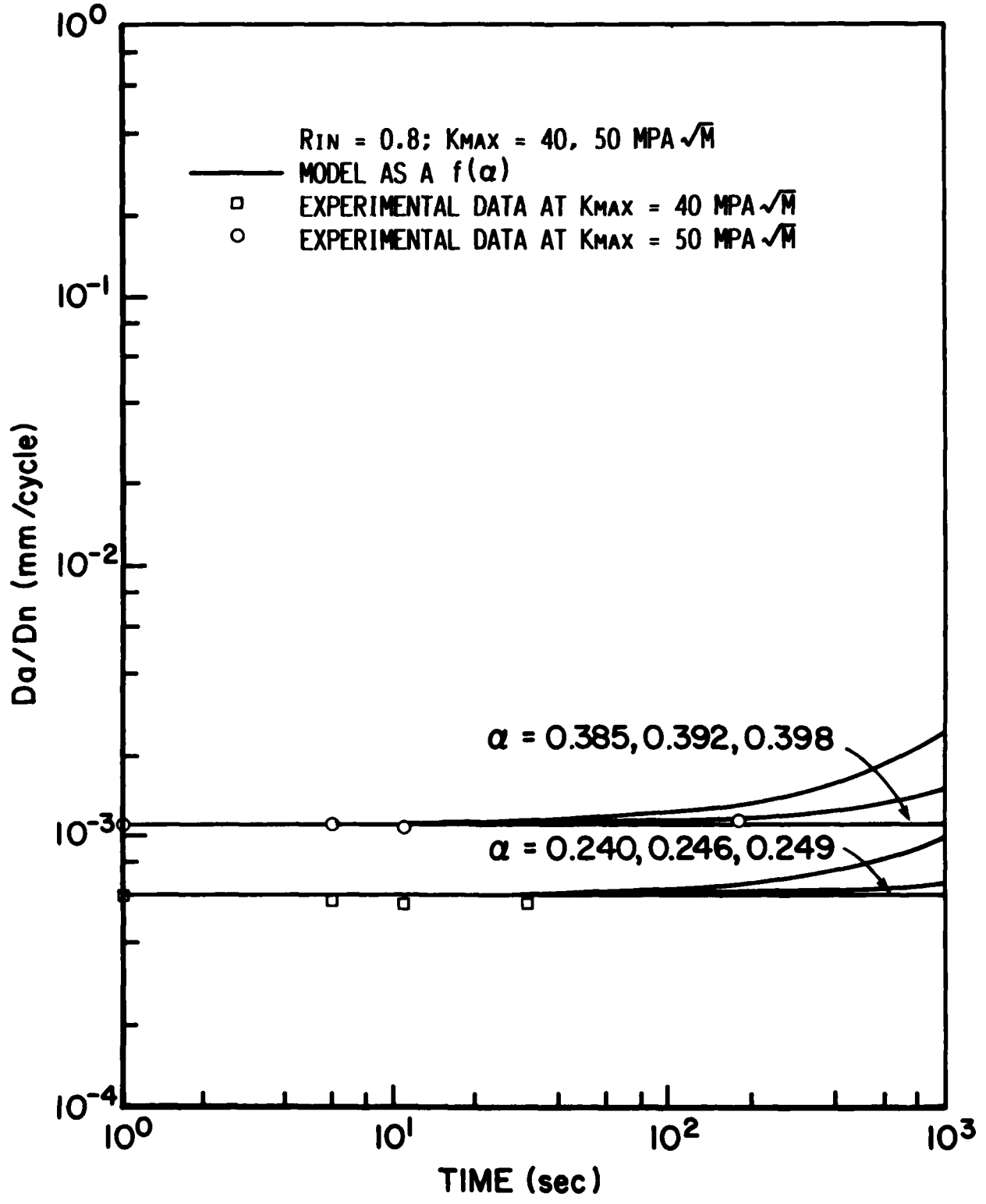


Figure 8. Calculated crack growth rate for various α for R_{in} = 0.8. Data are also shown in the figure.

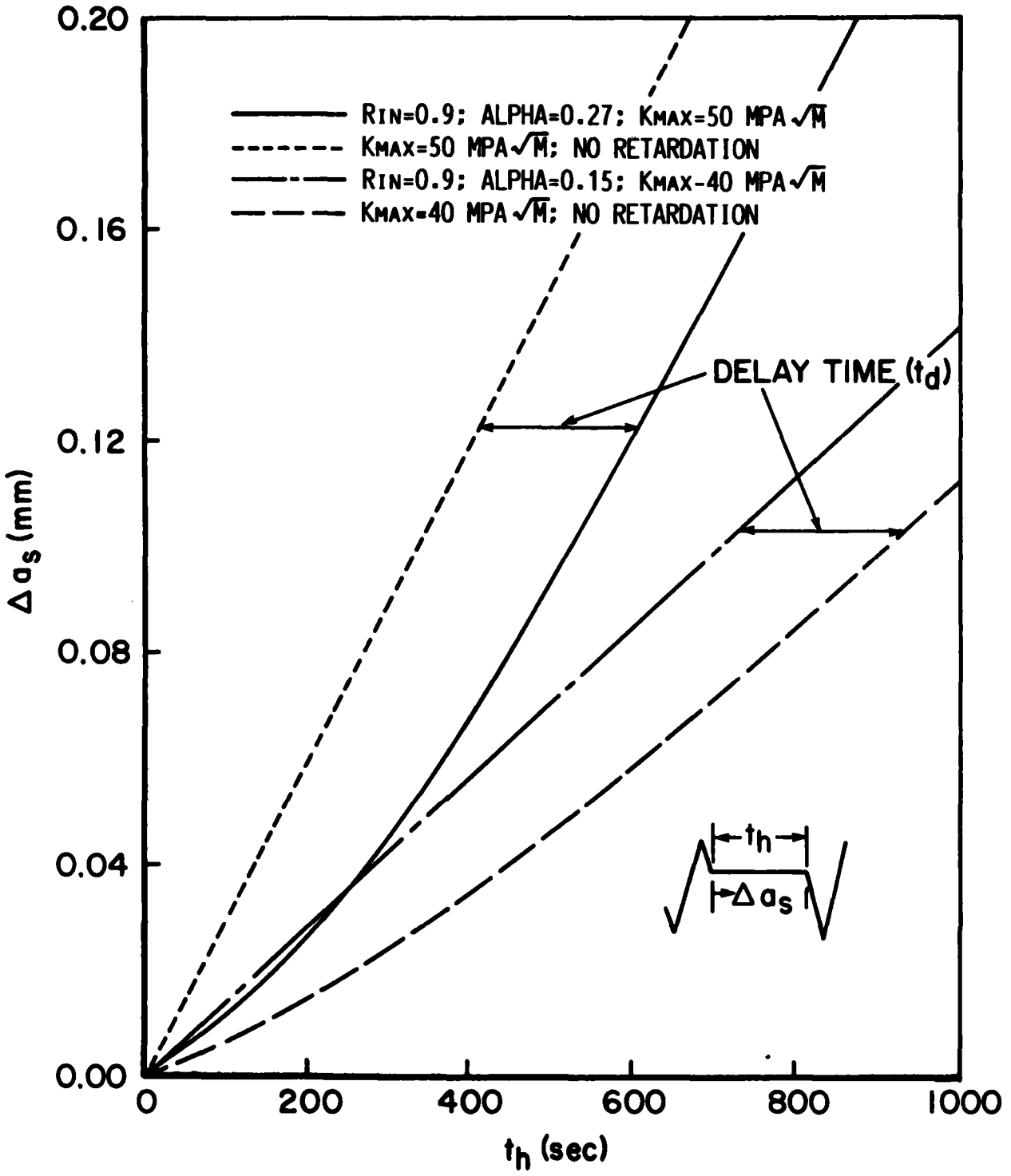


Figure 9. Calculated crack advance during sustained load after the application of the overload as a function of the sustained load time for R_{in} = 0.9 and K_{max} = 40 and 50 MPa·m$^{\frac{1}{2}}$ as predicted from the retardation model. Predicted delay times (t_d) are also shown in the figure.